HIGH ENERGY INTERACTION WITH THE NUCLEUS IN THE PERTURBATIVE QCD WITH $N_c \to \infty$

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The BFKL fan diagram equation for the scattering on the nucleus is solved numerically with the eikonalized initial condition and for a realistic nuclear density. The gluon density has a soliton-like form in the $\log q - y$ space. Inclusive cross-sections for jet production in hA and AB collisions are calculated.

1 Introduction

In the colour dipole approach with $N_c \to \infty$ [1,2] the interaction with a heavy nucleus is exactly described by a sum of fan diagrams constructed of BFKL pomerons, each of them splitting into two [3,4]. The equation for this sum has been constructed in [4,5,6]. Let the forward scattering amplitude on the nucleus at fixed impact parameter b be

$$\mathcal{A}(y,b) = 2is \int d^2r \rho(r) \Phi(y,b,r). \tag{1}$$

where ρ is the colour density of the projectile and Φ represents the sum of all BFKL fan diagrams attached to it. Then function $\phi = \Phi/(2\pi r^2)$ in the momentum space satisfies a non-linear evolution equation in the rapidity

$$(\partial/\partial y + H)\,\phi(y,q,b) = -\phi^2(y,q,b), \tag{2}$$

where $y = Y/\bar{\alpha}$ with $\bar{\alpha} = \alpha_s N_c/\pi$ is a rescaled rapidity and $H = \ln r^2 + \ln q^2 + const$ is the forward BFKL Hamiltonian. Eq. (2) should be solved with an initial condition which follows from the Glauber form of Φ at zero rapidity:

$$\Phi(0,r,b) = 1 - \exp\Big(-8\pi^2 AT(b) \int d^2r' G(0,r,r')\rho_N(r')\Big). \tag{3}$$

Here G is the forward BFKL Green function, T is the nucleus profile function and ρ_N is the colour density of the nucleon from the target nucleus. Eq. (2) was studied perturbatively in [7], by asymptotic estmates in [8] and finally solved numerically in [6] under the simplifying assumptions that the nucleus has a finite radius and that multiple Glauber interactions in (3) can be neglected, which is justified if $A^{1/3} << N_c$.

At small q the solution of (2) behaves like $const - \log q$ and consequently $\Phi(y, r, b) \to 1$ at $y \to \infty$ and fixed r and b. This corresponds to the saturation at high rapidities of the cross-section for the scattering of a colour dipole on the nucleus to its unitary black disk limit. However this does not imply that the structure function of the nucleus saturates, since the number of colour dipoles generated by the virtual photon is unlimited. In fact, as argued in [7,8] and confirmed by numerical calculations in [6], it grows with y. The gluon density of the nucleus in the combined momentum-impact parameter space

$$\partial xG(x, q, b)/\partial^2 q \partial^2 b = \partial^2 \phi/(2\pi\bar{\alpha}(\partial \ln q)^2) \equiv h/(2\pi\bar{\alpha}), \quad y = -\ln x \qquad (4)$$

was found to be a soliton wave in $y - \ln q$ space moving towards higher rapidities with a constant velocity and preserving its nearly Gaussian shape. A fit to numerical data with $\xi = \ln q$ gives

$$h(y, q, b) = h_0 e^{-a(\xi - \xi_0(y,b))^2}, \quad \xi_0 = -3.11 + (2/3)\ln B + \Delta_0 y, \qquad (5)$$

where $\Delta_0 = 2.3\bar{\alpha}$, $h_0 \simeq 0.3$ and $a \simeq 0.3$ are universal. Dimensionless parameter $B = \pi\alpha_s^2 AT(b)R_N^2$ where R_N is the nucleon radius. Actually the asymptotics of h at high $|\xi|$ is not Gaussian but exponential: at $\xi \to \infty$ $h \sim \exp(-1.3\xi)$ and at $\xi \to -\infty$ $h \sim \exp 2\xi$.

Here we first report on the improved numerical solution of Eq. (2) which takes into account multiple Glauber collisions in (3) and a realistic form of nucleus. Next we consider simplest of the production processes: the single inclusive jet production in hA and AB collisions and double inclusive jet production in hA collisions.

2 Improved nucleus structure function and gluon densities

Our first improvement consists in taking into account the full Glauber series in the initial value (3). Since at large b and consequently small $T(b)$ the contribution of the multiple scattering is neglegible, we used the same simplified profile function corresponding to a finite nucleus as in [6]. It was noted in [6] that the solution of (2) very quickly forgets its initial form as y grows, so that one expects to notice the influence of the glauberization of the initial function only at the initial stage of the evolution. This is confirmed by numerical calculations. The gluon densities obtained from the full initial function (3) and from its single rescattering term only differ at small y and q, where the former tends to zero and the latter tends to a constant. This difference turns out to be completely washed out already at $y = 1$. For $y \geq 1$ the two gluon densities are identical for all values of q. So our first conclusion is that the

Glauber rescattering at small y has practically no influence on the behaviour of the interaction with the nucleus at high rapidities.

Next we studied the contribution of very peripheric collisions for realistic nuclei with exponentially falling profile function. The interest in this effect is due to the fact that this contribution is not damped by the non-linear term and grows exponentially with y until $\phi \sim 1$. If $T(b) \sim \exp(-b/R_A)$ then this happens at $b \sim b_0 = R_A \Delta y$ where $\Delta = 4 \ln 2$ is the BFKL intercept. The total cross-section for the scattering of a colour dipole on the nucleus at $b > b_0$ and $y >> 1$ then grows linearly with y. Corespondingly one expects that the structure function will grow with y faster than linearly. These expectations are also confirmed by our numerical calculations. At large values of $1/x = \exp Y$ ($\sim 10^{15} \div 10^{20}$) the found structure functions of Pb with a constant density inside a finite sphere (CD) and with the Woods-Saxon nuclear density (WS) can be fitted by

$$F_2^{CD} \simeq Q^2\,(332y - 73 \ln Q^2 - 893),$$
$$F_2^{WS} \simeq Q^2\,(12.4y - 2.58 \ln Q^2 - 29.0)^2,$$

where $y = \bar{\alpha} Y$ and Q^2 is in $(\mathrm{GeV/c})^2$. Thus, to a good approximation, the structure function of a realistic nucleus grows with $1/x$ as $\ln^2(1/x)$.

Finally a few words about our definition of the gluon density in relation to the saturation phenomenon as discussed by A.Mueller [9]. We define the gluon density as essentially the quantity to be integrated with the quark loop to obtain the virtual photon scattering cross-section and structure function in the low x kinematical region. This definition is appropriate for this kinematical region, without strong ordering in the transverse momentum, and cannot be directly applied to the large x region where such ordering takes place. This circumstance has to be taken into account when comparing our gluon density with differently defined ones. In particular the gluon density introduced in [9] via the interaction of a "gluonic current" with a nucleus is a totally different quantity, more appropriate for the large x region. One can show that A.Mueller's density can be expressed in terms of our density (4) and that its saturation properties obtained in [9] follow from the found solitonic form of (4) [10].

3 Single jet inclusive production

Single jet production inclusive cross-sections in hA collisions are obtained from the imaginary part of forward scattering amplitude $\mathcal{A}$ (Eq. (1)) by a substitution of one of the BFKL Green functions in the fan diagrams

$$G(Y) \to G(Y-y) V_k(r) G(y), \quad V_k(r) = (4N_c \alpha_s / k^2) \overleftarrow{\Delta}\, e^{ikr} \overrightarrow{\Delta} \qquad (6)$$

(the arrows shows the direction of differentiation). Due to the AGK rules contribution of all such substitutions below the uppermost spliiting point cancel so that we are left with

$$d\sigma/dyd^2kd^2b = 2\langle\rho G(Y-y)V_k\Phi(y)\rangle = (16\pi^2\bar{\alpha}/k^2)\langle\rho[G(Y-y)\overleftarrow{\Delta}]e^{ikr}h(y)\rangle, \tag{7}$$

where ρ and Φ are from Eq. (1), $\langle ...\rangle$ means integrating over the gluon relative transverse coordinates or momenta and the relation between Φ and ϕ and the explicit form of V_k have been used to obtain the second equality.

Using the form of h obtained numerically we find jet multiplities at large y and $Y-y$

$$\mu_A(Y,y,k) = (c/k^2)A^{2/9}e^{\Delta Y-\epsilon y}/\sqrt{y(Y-y)} \tag{8}$$

where $\Delta = 4\ln 2$, $\epsilon = 0.39$, c is a known numerical constant and all rapidities are scaled with $\bar{\alpha}$. The inclusive cross-section integrated over k evidently diverges at low k^2. Physically relevant results correspond to jets with not too small transverse momentum $k > k_{min}$.

The A dependence is different both from the eikonal and local pomeron fan diagrams predictions. The k dependence is $1/k^2$ up to $\ln k \sim \Delta y$ in contrast to the hh scattering where it is cut by the damping factor $\exp(-c\ln^2 k/y)$. In this way we observe the "Cronin effect": the distribution for the nucleus target is flatter than for the hadronic one.

These results are easily generalized to nucleus-nucleus scattering. As was shown in [11], for them the single inclusive cross-section is given by the sum of fan diagrams connecting the produced jet with the both participant nuclei. So at fixed impact parameters of the nuclei b_A and b_B the inclusive cross-section is given by an obvious generalization of Eq. (7):

$$d\sigma/dyd^2kd^2b_Ad^2b_B = 2\langle\Phi_A(Y-y)V_k\Phi(y)\rangle = (32\pi^3\bar{\alpha}/k^2)\langle h_A(Y-y)e^{ikr}h_B(y)\rangle \tag{9}$$

Using our results for the gluon densities, for $\ln k << \xi_0$, $y \sim Y/2$ and identical nuclei, we obtain rough estimates $\sim (1/k^2)A^{4/9}e^{\Delta_1 Y}$ for the inclusive cross-section at a given k and $\sim A^{2/3}e^{\frac{3}{2}\Delta_1 Y}$ for the cross-section integrated over $k > k_{min}$ with $\Delta_1 = 2.4$. Note that from the latter estimate it follows that the multiplicity density at a given y behaves as $A^{4/3}$, that is, similarly to the eikonal result.

These estimates are supported by our numerical calculations. For Pb-Pb collisons at center rapidity with $\alpha = 0.16$ we obtained values for the multiplicity which are well described by a fit

$$\mu(0) = 0.092\, e^{0.59Y}\,(1 + 35.e^{-0.15Y}).$$

Note that with $\alpha = 0.16$ the BFKL pomeron intercept is $\Delta = 0.42$, so that the multplicities grow faster than the BFKL pomeron. The A dependence of $\mu(0)$ at a given Y is almost perfectly represented by a power factor $\mu(0) \sim A^{\beta(Y)}$, the power $\beta(Y)$ slowly rising with Y from 1.25 at $Y = 10 \div 20$ to 1.29 at $Y = 40$. So it seems that the difference in the A dependence with the eikonal prediction ($\beta = 4/3$) is gradually disappearing at high enough rapidities.

The form of the distribution in y turns out to be practically independent of the atomic number. With the growth of energy the distribution becomes more strongly peaked at the center.

4 Double inclusive cross-sections

For the nucleus-nucleus collisions the double inclusive cross-sections cannot be expressed via sums of fan diagrams for the participant nuclei, but require summation of diagrams of a more complicated structure [11]. For this reason we restrict ourselves to the case of hA scattering. The double inclusive cross-section is obtained from the imaginary part of the amplitude (1) by two substitutions (6) for the two produced jets with rapidities $y_{1,2}$ and transverse momenta $k_{1,2}$. The AGK rules leave two contributions corresponding to the two substitutions either in the uppermost pomeron or in the two pomerons immediately after the first splitting. These two contributions rise with the overall rapidity Y as $e^{\Delta Y}$ and $e^{2\Delta Y}$ respectively, so that the second one is dominant at high Y.

Study of this contribution shows that the multiplicity ratio

$$R = \frac{\mu(y_1, k_1; y_2, k_2)}{\mu(y_1, k_1)\mu(k_2, y_2)} = c\sqrt{\frac{(Y - y_1)(Y - y_2)}{2Y - y_1 - y_2}}, \tag{10}$$

where c is a known constant proportional to α_s, does not depend on $k_{1,2}$, grows with Y at center rapidity and stays constant in the diffractive regions. With $\alpha_s = 0.2$ at $y = Y/2$ we find $R = 0.19\sqrt{Y}$ so that correlations change from negative to positive at $Y \sim 25$.

5 Conclusions

Improved numerical calculations confirm the validity of approximations made in solving Eq. (2) in [6]. The nuclear structure function seems to grow with rapidity also for a realistic nucleus with a profile function exponentially falling at large impact parameters, the growth being faster than for a finite nucleus.

The inclusive jet production cross-sections grow rapidly with energy as the pomeron itself or even faster (for AA collisions). Their A- dependence results

close to the eikonal predictions in evident contrast to the earlier ones made for fan diagrams with a local pomeron.

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References

1. A.Mueller,*Nucl. Phys.* B **415**, 373 (1994).
2. A.Mueller and B.Patel,*Nucl. Phys.* B **425**, 471 (1994).
3. M.A.Braun and G.P.Vacca, Eur. Phys. J. C **6**, 147 (1999).
4. Yu. Kovchegov, *Phys. Rev.* D **60**, 034008 (1999).
5. I.Balitsky, *Nucl. Phys.* B **463**, 99 (1996).
6. M.A.Braun, Eur. Phys. J. C **16**, 337 (2000).
7. Yu. Kovchegov, *preprint CERN-TH/99-166 (hep-ph/9905214).*
8. E.Levin and K.Tuchin, *preprint DESY 99-108, TAUP 2592-99 (hep-ph/9908317).*
9. A.H.Mueller, *hep-ph/9904404.*
10. M.A.Braun, *hep-ph/0010041.*
11. D.Amati, L.Caneshi and R.Jengo, NPB **101**, 397 (1975).